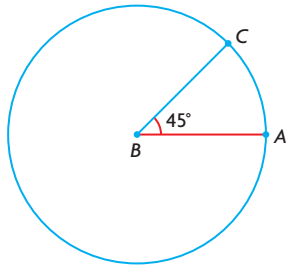
Chapter 8 – Sinusoidal Functions

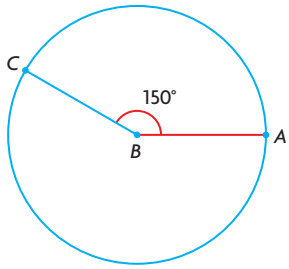
Lesson 8.1, page 489

1. **a)**



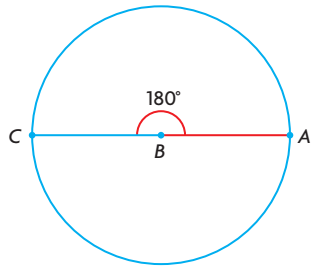
0.8

b)



2.5

c)

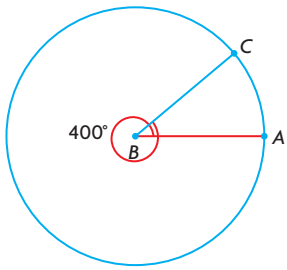


3.0

2. **a)** 90°
b) 30°
c) 135°
d) 270°

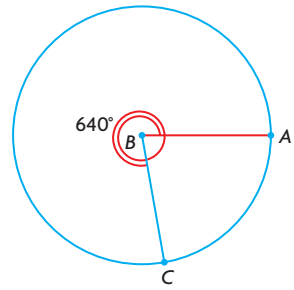
3. **a)** 60°
b) 235°
c) 390°

4. **a)**



7.0

b)



11.0

5. **a)** 450°
b) 600°

6. **a)** **i)** 9:20 a.m.
ii) 9:55 a.m.
iii) 10:55 a.m.
b) **i)** 2.1
ii) 5.8
iii) 12

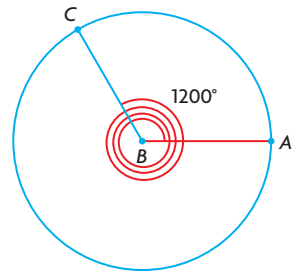
7. **a)** about 6.28 m
b) 2, 120°

8. **a)** 2
b) 45°
c) 5
d) 400°

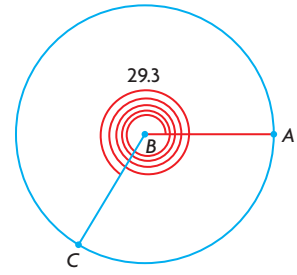
9. Disagree, e.g., For the circle with radius 5 m, the arc of an angle will be half as long as the arc of the equivalent angle in the circle of radius 10 m, but the ratio of arc length to radius, which gives the radian angle measure, will be the same.

10. e.g., Use benchmarks to estimate the radian measure equivalent of angles greater than 360°, where 1 radian is about 60°. For example, determine the radian measure of
i) 480° **ii)** 525° **iii)** 650°
i) $480° = 8 \cdot 60°$, and $8 \cdot 1 = 8$
OR $480° = 360° + 90° + 30°$, and $2\pi + \frac{\pi}{2} + \frac{\pi}{6} = \frac{13\pi}{6}$
ii) $525° < 540°$
$540° = 9 \cdot 60°$, and $9 \cdot 1 = 9$
OR $540° = 360° + 180°$, and $2\pi + \pi = 3\pi$
iii) $650° < 660°$
$660° = 11 \cdot 60 = 11$
OR $660° = 360° + 180° + 120°$, and $2\pi + \pi + \frac{\pi}{3} = \frac{10\pi}{3}$

11. **a)**



b)



Answers

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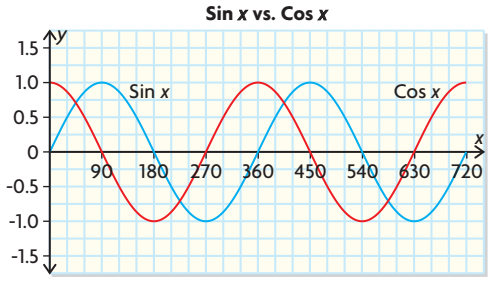
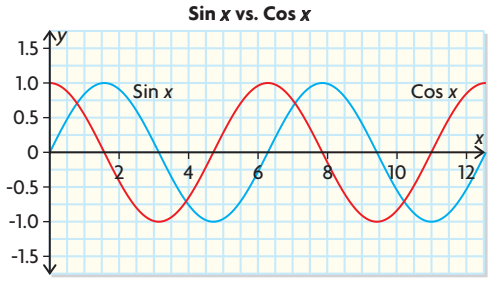
12. **a)** 4.0
b) 3.0
c) 5.9

Lesson 8.2, page 494

1. **a)**



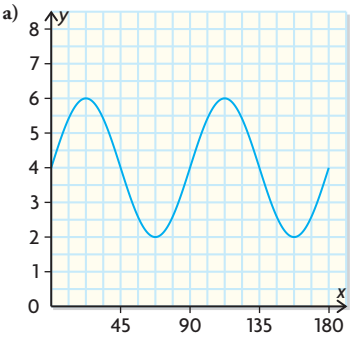
b) 0; 180° and 540°
c) 0; 270° and 630°
d)



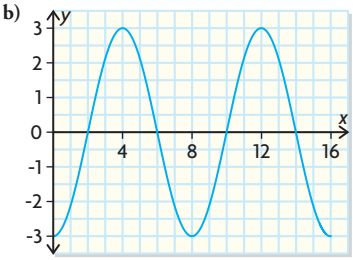
2. e.g., The graph of $y = \sin x$ repeats itself after it passes through 360° or 2π.
3. e.g., The graph is symmetrical along the x-axis, with the axis of symmetry being at 90° and 270°, respectively. The graph is rotationally symmetrical around the point (180°, 0).
4. e.g., The graph is symmetrical along the x-axis, with the axis of symmetry being at 180° and 360°, respectively. The graph is rotationally symmetrical around the point (270°, 0).
5. **a)** 0
b) 1
6. **a)** 0°, 180°, 360°, 540°, 720°
b) 90°, 270°, 450°, 630°
7. e.g., The up-and-down pattern in the graph of $y = \sin x$ looks like the repeating pattern of waves in a lake.
8. e.g., Yes, because each point on the sine graph corresponds to a point on the cosine graph with the same y-coordinate but with an x-coordinate of 90° less.
9. e.g., It would look like the graph of cos x, as it would start at 1, decrease to 0 after spinning 90°, then decrease to -1 after spinning 180°, and so on.

Lesson 8.3, page 506

1. **a)** $\{y \mid -3.5 \le y \le 3.5, y \in \mathrm{R}\}$, 3.5
b) $\{y \mid -3 \le y \le 1, y \in \mathrm{R}\}$, 2
2. **a)** $y = 2$, 1.5
b) $y = -1$; 4
3. **a)** 120°
b) 3
4. **a)** $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$, 5, $y = -2$, 180°
b) $\{y \mid -0.5 \le y \le 6.5, y \in \mathrm{R}\}$, 3.5, $y = 3$, 5
5. $\{y \mid -2 \le y \le 3, y \in \mathrm{R}\}$, 2.5, $y = 0.5$, 4.25
6. **a)**



b)



7. e.g., The period would be the same, but the amplitude (and the minima and maxima) would be smaller.
8. **a)** 1.6 m
b) 0.8 m
c) 2.5 s
d) about 1 m, 1.6 m
9. **a)** $y = 0$; e.g., velocity of the air between breaths.
b) 0.8 L/s
c) 5 s; e.g., time to breathe in and out completely
10. **a)** e.g., Both: the increased amplitude means that the breaths are deeper and the decreased period means more breaths per minute.
b) e.g., amplitude and period have changed
c) 1.6 L/s
11. e.g., The gymnast completes a jump every 3 seconds, going from 3 feet above the ground to 30 feet, with a total jump height of 27 feet.
12. range: $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$; amplitude: 5; midline: $y = -2$; period: 0.5
13. **a)** A: period = 1 s, minimum = 10 cm, maximum = 26 cm, amplitude = 8 cm;
B: period = 0.5 s, minimum = 10 cm, maximum = 18 cm, amplitude = 4 cm
b) e.g., A, because its amplitude is greater.
14. **a)** e.g., hoop's radius
b) hoop 3
c) hoop 1; hoop 3 or hoop 2
d) e.g., hoop 3, because it is smaller, but is being spun at the same rate as hoop 2
15. e.g., The period and midline stay the same, while the amplitude decreases. e.g., The range is the difference of the maximum and minimum y-values. The amplitude is half the difference of the maximum and minimum values. The equation of the midline is y = average of the minimum and maximum values.

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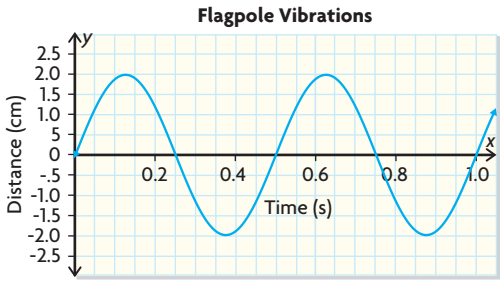
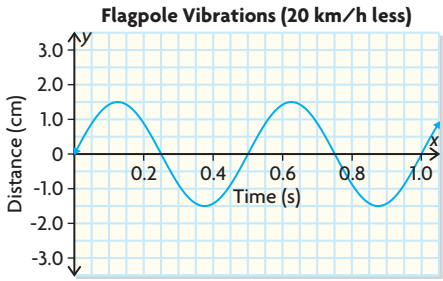
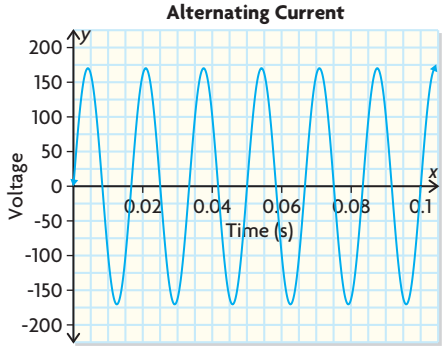
16.





17. **a)** graph 2, graph 1, graph 3
b) e.g., Each A pitch has double the frequency of the previous one.
c) graph 2: 110 Hz, graph 3: 440 Hz
d) e.g., Sound waves can be approximated by a sinusoidal function. The volume of the sound is dictated by the amplitude. The period, or frequency, varies with the size of the instrument (e.g., a tuba creates low-frequency/long-period waves while a piccolo creates high-frequency waves.)

18. **a)** about 17 ms
b)



Mid-Chapter Review, page 515

1. **a)** 2.4
b) 3.6
c) 7.0
d) 9.0

2. **a)** 60°
b) 180°
c) 360°
d) 540°

3. 0.1, 20°, 0.5, 1.5, π, 190°, 200°, 310°, 400°

4. **a)** $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$, 1, 360° or 2π, $y = 0$
b) e.g., Their range, amplitude, period and equation of midline are the same; their y-intercept is different.

5. **a)** 0°, 180°, 360°, 540°, 720°
b) 90°, 450°: 270°

6. **a)** $\{y \mid -5 \leq y \leq 1, y \in \mathrm{R}\}$, 3, 180°, $y = -2$
b) $\{y \mid -2.5 \leq y \leq 1.5, y \in \mathrm{R}\}$, 2, 360°, $y = -0.5$

7. **a)** range: $\{y \mid -4 \leq y \leq 6, y \in \mathrm{R}\}$, amplitude: 5, equation of midline: $y = 1$, period: 7
b) range: $\{y \mid 0.5 \leq y \leq 3.5, y \in \mathrm{R}\}$, amplitude: 1.5, equation of midline: 2, period: 0.5

8. **a)** $\frac{1}{3}$ s
b) $y = 0$ amperes
c) 4.5 amperes

Lesson 8.4, page 528

1. c), a), b); e.g., I ordered the functions based on the values of a.

2. c), a), b); e.g., The range is defined by the function's amplitude, so I ordered the functions based on the values of a.

3. a), b), c); e.g., The magnitude of the period is inversely proportional to b.

4. **a)** 45° left
b) 180° right
c) 45° right

5. **a)** 7, $\{y \mid -7 \leq y \leq 7, y \in \mathrm{R}\}$
b) 13, $\{y \mid -13 \leq y \leq 13, y \in \mathrm{R}\}$

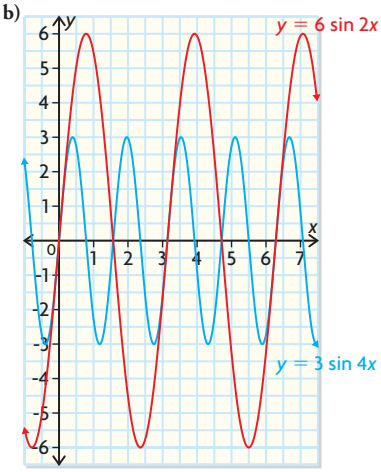
6. **a)** $y = 5$, 8, $\{y \mid -3 \leq y \leq 13, y \in \mathrm{R}\}$
b) $y = -7$, 6, $\{y \mid -13 \leq y \leq -1, y \in \mathrm{R}\}$

7. **a)** $y = 2$; 7; -3
b) $y = -3$; 0; -6

8. **a)** 30° right
b) 100° left
c) 4.5 right
d) 3 left

9. **a)** 90°; 45° right
b) 4π; 1 right

10. e.g.,
a) $y = 6 \sin 2x$
b)



Answers

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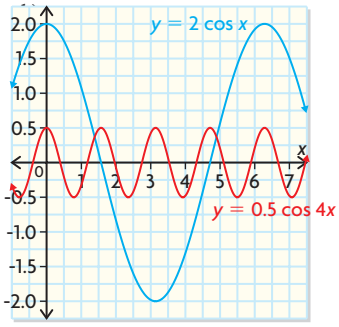
11. e.g.,

a) $y = 0.5 \cos 4x$

b)



12. **a)** 86 ft

b) about 1.8 min, or 1 min 48 sec

c) 4 ft

13. **a)** iii

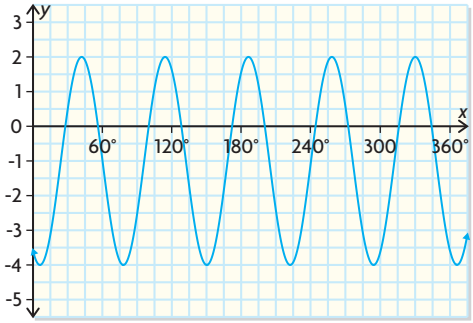
b) i

14. **a)** v

b) ii

15. **a)** e.g., sinusoidal function, amplitude: 3, midline: $y = -1$, range: $\{y \mid -4 \leq y \leq 2, y \in \mathrm{R}\}$, period: 72°, horizontal translation from $y = \cos x$: 30° left

b)



16. **a)**

Note	Period (s)	Frequency (Hz)
G	0.0051	196.0
D	0.0034	293.9
A	0.0023	440.2
E	0.0015	659.6

b) e.g., Each string is tuned to almost 1.5 times the frequency of the previous string.

c) e.g., About 2 periods of the lower note and 3 periods of the higher note. The frequency of the higher note is approximately 1.5 times the frequency of the lower note.

d) e.g., 4, 6, and 9 periods

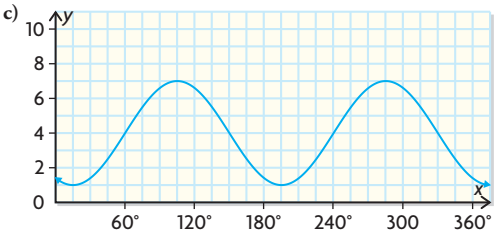
17. **a)** **i)** 3, $y = 4$, $\{y \mid 1 \leq y \leq 7, y \in \mathrm{R}\}$, 180°, 60° to the right

ii) 3, $y = 4$, $\{y \mid 1 \leq y \leq 7, y \in \mathrm{R}\}$, 180°, 240° to the right

iii) 3, $y = 4$, $\{y \mid 1 \leq y \leq 7, y \in \mathrm{R}\}$, 180°, 120° to the left

b) e.g., The graphs are all the same.

c)



e.g., It is also the same because it is a cosine graph with the same attributes as the sine graphs, except that it is translated to the left by 45° (90° divided by b).

18. **a)** 10.5 cm, 2.5 cm

b) 0.25 s, e.g., The apple completes 4 bounces per second.

19. **a)** 33 m, 3 m

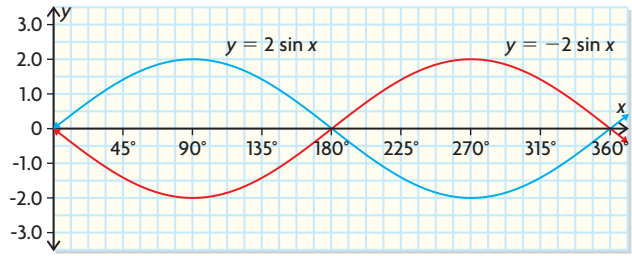
b) 4π min or about 12.6 min, e.g., The wheel completes a rotation every 12.6 minutes.

20. **a)** 0.75 s

b) e.g., A person's blood pressure makes a complete low-to-high cycle about every 0.75 seconds.

21. e.g., If the equation is in the form $y = a \sin b(x + c) + d$, the amplitude is a, the midline is $y = d$, it is translated to the right by c degrees or radians, and the period is $\frac{360°}{b}$ or $\frac{2\pi}{b}$. For example, the equation $y = 2 \sin 2(x + 45°) + 3$ has an amplitude of 2, a midline of $y = 3$, it is translated to the right 45°, and a period of 180°.

22. **a)**



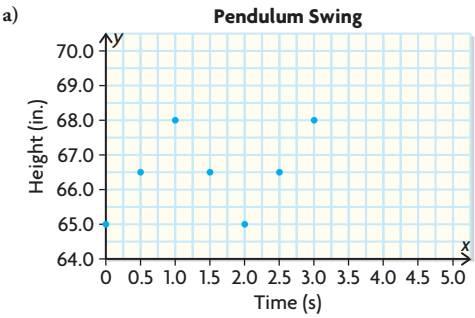
e.g., The period, amplitude and equation of midline are the same; they are reflected across the x-axis.

b) e.g., A translation of π or 180°.

c) Yes, e.g., graphs of functions of the form $y = a \sin x$ and $y = -a \sin x$ are horizontal translations of each other by 180° or π.

Lesson 8.5, page 541

1. **a)**



b) $y = 1.5 \sin(3.141\ldots x - 1.570\ldots) + 66.5$

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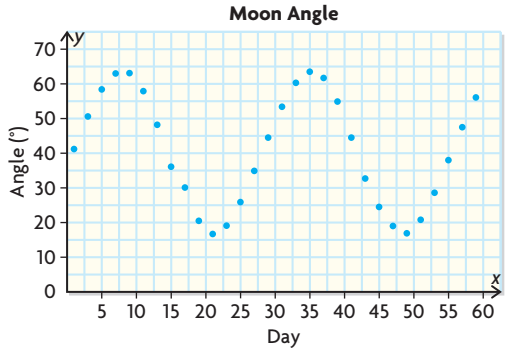
2. **a)**

Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
Height (ft)	11	6	1	6	11	6	1	6	11

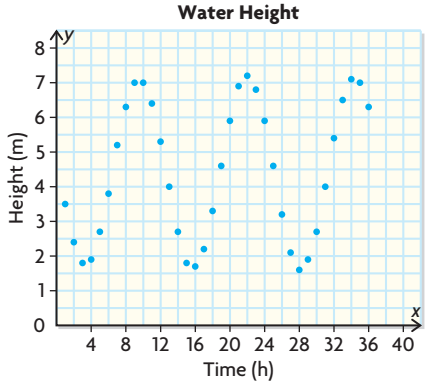
b) $y = 5 \sin (3.141\ldots x + 1.570\ldots) + 6$
c) 1 ft 7 in.
d) 1 ft 11 in.

3. **a)**



e.g., about 28 days
b) e.g., Because the data follow a sinusoidal pattern.
c) $y = 22.954\ldots \sin (0.230\ldots x - 0.208\ldots) + 40.284\ldots$
d) 63.2°
e) 23.8°
f) May 14

4. **a)** e.g., Because the data follows a sinusoidal pattern.
b)



$y = 2.726\ldots \sin (0.505\ldots x + 3.016\ldots) + 4.426\ldots$; yes
c) 7.2 m, 1.7 m
d) 12 h 26 min
e) 4.4 m

5. 6 h 48 min

6. **a)** $y = 19.581\ldots \sin (0.462\ldots x - 1.662\ldots) + 6.238\ldots$
b) $y = 16.992\ldots \sin (0.470\ldots x - 1.751\ldots) - 4.738\ldots$
c) $y = 18.388\ldots \sin (0.464\ldots x - 1.694\ldots) + 0.686\ldots$
d) e.g., The amplitudes, phase shifts and periods are similar; the equations of the midlines are different.
e) e.g., Yes, but it is more difficult to fit a curve to it.
f) 14.1°C

7. **a)** $y = 20.341\ldots \sin (0.536\ldots x - 2.332\ldots) - 2.706\ldots$
b) 17.6°C, −23.0°C; e.g., the function is reasonably accurate.
c) 16.1°C

8. **a)** $y = 4.207\ldots \sin (0.017\ldots x - 1.398\ldots) + 12.411\ldots$
b) 8 h 12 min to 16 h 37 min
c) day 171
d) 9 h 10 min
e) days 119 and 224

9. **a)** 4.7°C, e.g., I determined a sinusoidal regression function that models the data, substituting numbers for months, and extrapolated the predicted value for the tenth month.
b) 21 mm, e.g., I determined a sinusoidal regression function that models the data, substituting numbers for months, and extrapolated the predicted value for the tenth month.
c) e.g., Temperature, since rainfall is more likely to vary.

10. **a)**

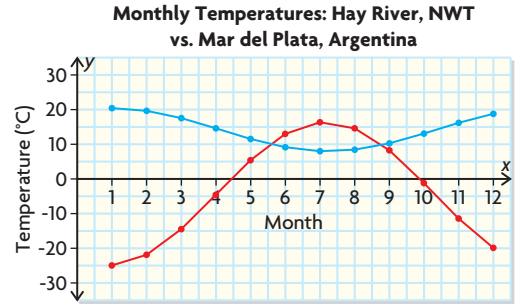
Time (s)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
Height (cm)	70	55	40	55	70	55	40	55	70	55	40	55	70

b) $y = 15 \sin (2\pi x + 1.570\ldots) + 55$
c) 55 cm

11. **a)** $y = 6 \sin (\pi x - 1.570\ldots) + 11$
b) 15.2 in., 14.5 in.

12. e.g., No, because the regression function takes into account the effects of other months.

13. **a)** Hay River: $y = 20.681\ldots \sin (0.501\ldots x - 2.018\ldots) - 4.284\ldots$, yes;
Mar del Plata: $y = 6.221\ldots \sin (0.504\ldots x + 1.059\ldots) + 14.188\ldots$, yes
b)



e.g., The graphs have very similar periods; the amplitude is much less for Mar del Plata, but the midline is much greater; the maximum is only slightly greater for Mar del Plata; the maximums and minimums of the graphs are almost exactly opposite.
c) e.g., Their locations (north or south of the equator) affect when the temperature is high or low. Their distance from the equator affect the magnitude of the temperature changes.
d) e.g., Late May and late August, since that is when the graphs intersect.
e) e.g., south, as January and February temperatures are warmer than July and August temperatures.

14. **a)** e.g. Because the Earth is moving along its orbit, so the Moon must "catch up."
b) e.g., The period of the two graphs will be different (with the synodic being longer), but all other attributes will be the same.

Chapter Self-Test, page 549

1. **a)** 3.2
b) 8.0
c) 0.3
d) 6.0

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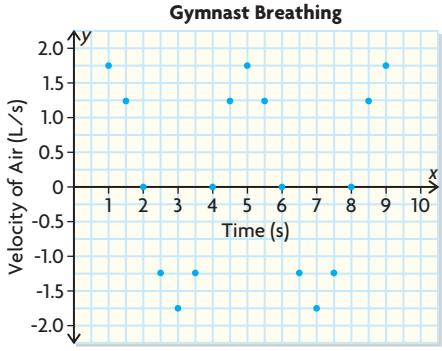
2. **a)** 315°
b) 270°
c) 330°
d) 540°

3. **a)** 0°, 60°, 120°, 180°, 240°, 300°, 360°, 420°, 480°, 540°, 600°, 660°, 720°
b) 0, 1.047, 2.094, 3.141, 4.189, 5.236, 6.283, 7.330, 8.378, 9.425, 10.472, 11.519, 12.566

4. **a)** $\{y \mid -1 \leq y \leq 7, y \in \mathrm{R}\}$, 4, 120°, 20° to the right, $y = 3$
b) $\{y \mid -6 \leq y \leq -2, y \in \mathrm{R}\}$, 2, 180°, 60° to the right, $y = -4$

5. $y = 19.557\ldots \sin(0.476\ldots x - 1.762\ldots) + 6.141\ldots$

6. **a)**



$y = 1.751\ldots \sin(1.570\ldots x)$
b) 4 s
c) e.g., Positive and negative velocities correspond to exhalations and inhalations (or vice versa).
d) 10 s, 12 s, 14 s, 16 s, 18 s

Chapter Review, page 551

1. **a)** 0.3
b) 2.0
c) 4.0
d) 9.6

2. **a)** 540°
b) 300°
c) 30°
d) 630°

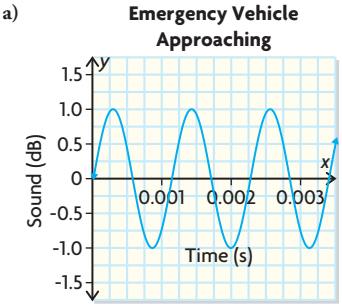
3. **a)** $180° < x < 360°$, $540° < x < 720°$
b) $0° \leq x < 90°$, $270° < x < 450°$, and $630° < x \leq 720°$

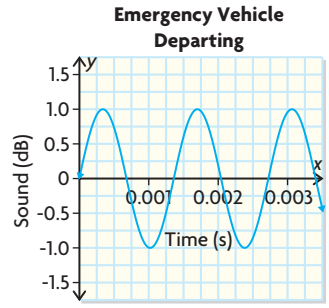
4. **a)** $\{y \mid -4 \leq y \leq 0, y \in \mathrm{R}\}$, 2, 360°, $y = -2$
b) $\{y \mid -2.5 \leq y \leq -0.5, y \in \mathrm{R}\}$, 1, 90°, $y = -1.5$

5. **a)** $y = 8$; e.g., She started at 8 m from the sensor.
b) 6 m
c) 4 s; this is the amount of time to complete one swing.
d) 2 m
e) e.g., Yes, since she is at the her furthest distance from the detector.

6. **a)** A: 3 m, 2 m, about 12 s; B: 4 m, 3 m, about 16 s
b) e.g., They are travelling at about the same speed (75 m in 48 s).

7. **a)**

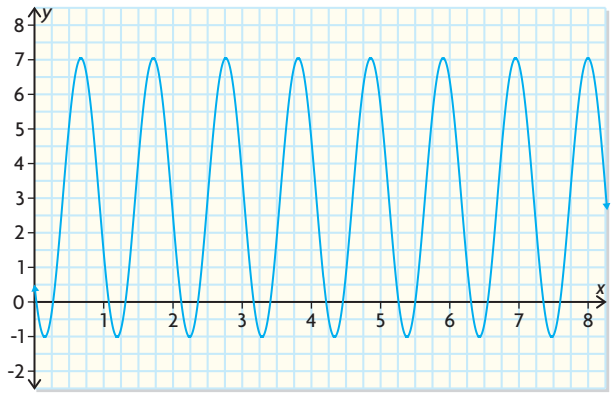




e.g., When it's driving toward you.
b) 877 Hz, 735 Hz

8. **a)** iii, iv
b) ii

9. **a)** 4, $y = 3$, $\{y \mid -1 \leq y \leq 7, y \in \mathrm{R}\}$, $\frac{\pi}{3}$, 2.5 right
b)



10. about 2.4 cycles per nanosecond; about 5.0 cycles per nanosecond
11. about −12.2°C
12. about 9.6°C

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Cumulative Review, page 556

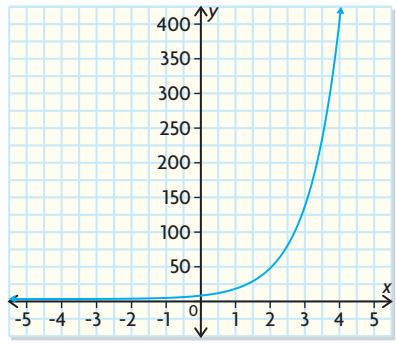
1. a) **i)** 0
ii) 2
iii) Curve extends from QII to QI.
iv) domain: $\{x \mid x \in R\}$
v) range: $\{y \mid y > 0, y \in R\}$
vi) increasing

b) **i)** 0
ii) 3
iii) Curve extends from QII to QI.
iv) domain: $\{x \mid x \in R\}$
v) range: $\{y \mid y > 0, y \in R\}$
vi) decreasing

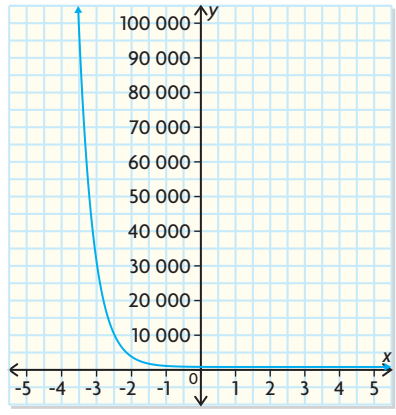
2.

Number of *x*-Intercepts	*y*-Intercept	End Behaviour	Domain	Range	Increasing or Decreasing?
0	5	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	increasing
0	30	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	decreasing

a)



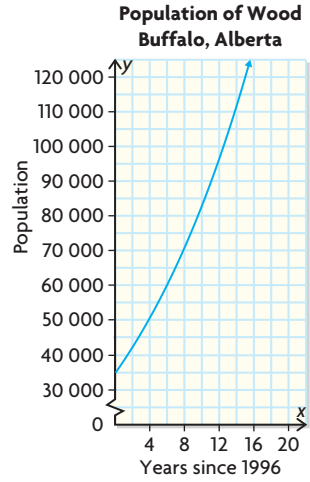
b)



3. a)

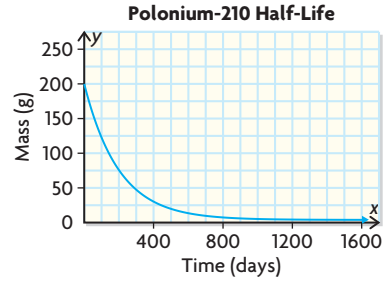
Year	Population
1996	35 000
1997	37 800
1998	40 824
1999	44 090
2000	47 617
2001	51 426
2002	55 541
2003	59 984
2004	64 783
2005	69 965
2006	75 562

b) e.g., The rate of growth is constant, and growth occurs rapidly.
c) 35 000; The *y*-intercept represents the initial population (in 1996).
d)



i) 111 025
ii) during 2009

4. a)



b) 0.02 g
c) 276 days

5. a) $x = 5$
b) $x = 10$
c) $x = 0.25$
d) $x = 6$

Answers

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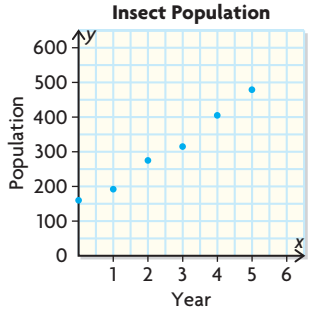
6. **a)** $x = 1.11$

b) $x = 0.53$

7. **a)**



b) $y = 161.58(1.252)^x$

c) e.g., a represents the initial population, 161.58, b represents the growth rate, 1.252.

d) after about 8 years

8. about 13.9 years

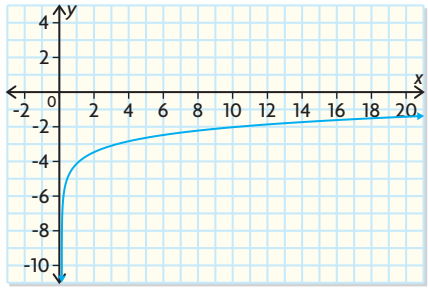
9. **a)** e.g., $y = 3.30$

b) e.g., $y = 6$

c) e.g., $y = -3$

d) e.g., $y = -1.48$

10. **a)** x-intercept: 1, y-intercept: none, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, function: increasing



b) x-intercept: 1, y-intercept: none, end behaviour: QI to QIV, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, function: decreasing

11. **a)** 9 **b)** -3 **c)** 10 **d)** 243 **e)** 0 **f)** 5

12. **a)** $\log_4 96$, 3.29

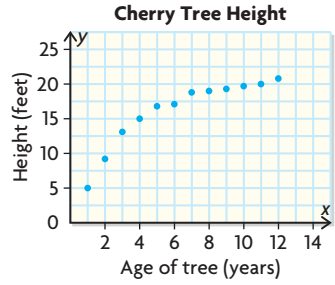
b) $\log_2 3$, 1.58

13. e.g., I can solve exponential equations by using the common logarithm and the change of base formula. I can bring down the exponent of an exponential expression by applying a logarithm of the same base as the exponential expression to both sides of the equation. Then I can use the change of base formula so that I can calculate the value of the logarithm using my calculator. For example,

$$\begin{aligned} 25 &= 4^x \\ \log_4 25 &= \log_4 4^x \\ \log_4 25 &= x \\ \frac{\log 25}{\log 4} &= x \\ 2.321... &= x \end{aligned}$$

14. about 8 years

15. **a)**



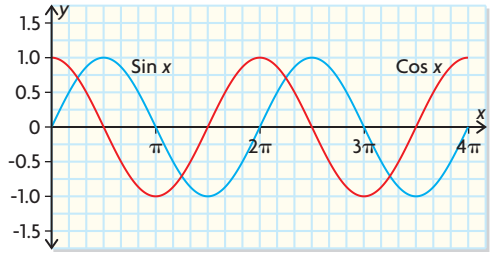
b) $y = 6.357... \ln x + 5.561...$

c) about 22.8 feet

d) about 2.8 years old

16. **a)** number of x-intercepts: multiple, y-intercept: $\sin x$, 0; $\cos x$, 1, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$, period: 2π; amplitude: 1, equation of the midline: $y = 0$.

b)



17. **a)** range: $\{y \mid 1 \leq y \leq 11, y \in \mathrm{R}\}$, midline: $y = 6$, amplitude: 5 m, period: 90 s

b) e.g., The Ferris wheel starts 1 m off the ground and over 90 s rotates to 11 m and back. The axis of the wheel is 6 m off the ground and the wheel has a radius of 5 m.

18.

	Amplitude	Period	Equation of the Midline	Domain	Range	Horizontal Translation
a)	3	90.5°	$y = 2$	$\{x \mid x \in \mathrm{R}\}$	$\{y \mid -1 \leq y \leq 5, y \in \mathrm{R}\}$	30° right
b)	5	6.29	$y = -2$	$\{x \mid x \in \mathrm{R}\}$	$\{y \mid -7 \leq y \leq 3, y \in \mathrm{R}\}$	2.28 right

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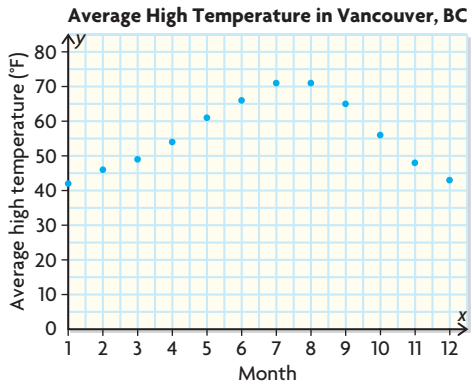
19. **a)**



e.g., The average monthly temperatures should be relatively consistent from year to year.

b) $y = 13.980... \sin(0.551...x - 2.366...) + 56.672...$

c) 70.6 °F

d) e.g., from late April to early October

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